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The TPACK model in action: towards an integrated system of professional knowledge of prospective teachers

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Abstract. *Introduction.* Effective preparation of prospective teachers requires developing a system of professional knowledge conceptually framed within the Technological Pedagogical Content Knowledge (TPACK) model. *Aim.* The present study *aims* to demonstrate the effectiveness of the TPACK model as a theoretical foundation for designing a university course that integrates evidence-based educational practices and fosters the development of a comprehensive system of professional competencies. *Methodology and research methods.* The study employs a combination of theoretical and empirical methods. A measurement tool, developed based on the TPACK model, was used alongside pre- and post-assessment procedures that included tasks from the subject, technological, and methodological domains. *Results and scientific novelty.* The methodological significance of the TPACK model lies in its role as a conceptual framework for analysing the integrative interaction among the disciplinary, methodological, and technological components of teacher education. This model provides a holistic approach to understanding how educational practices support the development of future teachers' professional knowledge. Additionally, principles for designing a university course that ensures the balanced development of subject matter, pedagogical, and technological knowledge are formulated. *Practical significance.* The experience of developing and implementing an elementary geometry course based on the TPACK model enhances the understanding of the unique characteristics of university-level teacher education and can be applied to other disciplines and areas within teacher education.

Keywords: TPACK model, teacher education, prospective teachers, integrative knowledge, educational practices

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Модель ТРАСК в действии: на пути к интегрированной системе профессиональных знаний будущих учителей

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Аннотация. Введение. Эффективная подготовка будущего учителя требует формирования системы профессиональных знаний, концептуально описываемой в рамках модели технологическо-педагогического предметного знания – ТРАСК (от англ. Technological Pedagogical Content Knowledge). Цель исследования – обоснование эффективности модели ТРАСК как теоретической основы построения университетского курса, интегрирующего проверенные образовательные практики и обеспечивающего формирование целостной системы профессиональных компетенций. Методология, методы и методики. Исследование опирается на комплекс теоретических и эмпирических методов. Применен измерительный инструментарий, разработанный на основе модели ТРАСК, а также процедуры входного и итогового контроля, включавшие задания предметного, технологического и методического блоков. Результаты и научная новизна. Обоснована методологическая значимость модели ТРАСК как концептуальной основы анализа интегративного взаимодействия дисциплинарного, методического и технологического компонентов педагогического образования. Доказано, что данная модель задает целостную рамку для понимания того, как образовательные практики опосредуют формирование профессиональных знаний будущих учителей. Сформулированы принципы проектирования университетского курса, обеспечивающие сбалансированное развитие предметных, педагогических и технологических знаний. Практическая значимость. Представленный опыт разработки и внедрения курса элементарной геометрии, ориентированного на модель ТРАСК, углубляет понимание специфики университетской подготовки будущих учителей и может быть экстраполирован на другие дисциплины и направления педагогического образования.

Ключевые слова: модель ТРАСК, педагогическое образование, будущие учителя, интегративные знания, образовательные практики

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Introduction

Teacher training at the university is aimed at the formation of knowledge of psychological and pedagogical patterns that determine the peculiarities of teaching and educating schoolchildren of different ages, mastering the fundamental foundations of the scientific field, which includes the discipline of the school course chosen by the student for prospective professional activities, deep and systemic knowledge directly on this discipline and the methodology of its teaching, as well as practical experience of their application. For example, a prospective mathematics teacher studies a whole range of fundamental disciplines (algebra and number theory, geometry, mathematical analysis, mathematical logic, probability theory and mathematical statistics, etc.), a cycle of psychological and pedagogical courses, as well as a number of disciplines focused on in-depth repetition and generalisation of the content of the school mathematics course, mastering it at a deeper and more systemic level.

Methodologically significant is the approach that views the preparation of prospective teachers as a process of integrating various types of professional knowledge – subject matter, pedagogical, and technological. This integration implies not merely a simple combination of these components but their mutual enrichment, resulting in a holistic understanding of the learning process as a unified whole encompassing content, methodology, and delivery tools. In this context, it is important to design university courses so that they foster an understanding of the relationships between what to teach, how to teach it, and the tools used to implement the educational process. The task of integrating various components of professional training is particularly important in courses focused on in-depth mastery and systematisation of the content of a school subject, which will subsequently form the basis of the prospective teacher's teaching activities.

The modern perspective on teacher training for the systematic mastery of both the content and methodological foundations of teaching any school subject involves addressing the following key tasks:

- to compensate and fill gaps in the knowledge and skills of students in relation to the content of the school course, while implementing two mutually inverse, but closely related processes of generalisation and concretisation of their ideas about the essence and structure of subject knowledge;
- to ensure that students master the fundamental ideas that are not fully disclosed in school textbooks, but represent the scientific basis of the school course;
- provide students with the opportunity to master the concepts, facts and methods that are the basis for the study of fundamental disciplines;
- to update the methodological component of the content of the school discipline;
- to form students' skills and abilities that determine productive subject and methodological activity, as well as to ensure the development of subject thinking at the level necessary for their successful work as a teacher; and

– to develop in preservice teachers a readiness to carry out educational and research activities as an effective form of knowledge, which is implemented using the means and methods characteristic of scientific activity and leads to the discovery of new facts related to the object of study.

Thus, university courses related to the in-depth study of the content of the school discipline play an important role in the preparation of the prospective teacher, since they are intended to become the foundation for the formation of not only subject knowledge as such, but also their methodological component. In this regard, an important area of research is the search for techniques, methods and technologies that would guarantee the achievement of all the above goals.

Obviously, the presented tasks can be easily tailored to the training of school teachers in any subject area. Thus, for example, while studying at university, future mathematics teachers must master several courses integrated into a cross-cutting curriculum of elementary mathematics. Z. A. Sarvanova [1] and L. V. Tymshina [2], agreeing on the targeted orientation of studying these disciplines, emphasise that their instruction should not be limited to problem-solving workshops alone. Instead, it should be based on the principle of integrativity, which aims to ensure the synthesis of both subject-specific and methodological knowledge for prospective teachers. In this regard, it is essential to consider the theory presented in the works of D. L. Ball, M. H. Tames, and G. Phelps [3], as well as H. Cheek and K. Beswick [4], which reflects the structure of mathematical knowledge for teaching (Mathematical Knowledge for Teaching – MKT). The nature and level of this knowledge should be such as to allow the teacher in the process of teaching mathematics not to broadcast the content of school textbooks, but to organise fruitful cognitive activities of students. Based on empirical studies by M. Hourigan and A. M. Leavy [5], L. V. Shkerina, M. B. Shashkina, and O. A. Tabinova [6], as well as O. V. Tumasheva, M. B. Shashkina, and E. A. Aleshina [7], it has been demonstrated that knowledge of the mathematical content of the school curriculum and its specialised form – mathematical content knowledge for teaching (MKT) – is often associated with professional deficiencies among prospective mathematics teachers. These findings also apply to the university training of teachers in other disciplines.

In the aspect of rapid technological progress, another important task of the university course of elementary geometry, which often remains in the shadows, is the formation of pedagogical knowledge among future teachers on the use of technologies in the subject context. The study of this problem led P. Mishra and M. L. Koehler [8] to develop an integrative model of TPACK (Technological Pedagogical Content Knowledge). This model describes the comprehensive systemic knowledge required for a school course, encompassing understanding of the methodological context of the subject matter as well as knowledge related to the use of technology for teaching specific content. Since the model takes into account the three main components of the educational process (subject content, methodological aspects of teaching and technological tools to increase its effectiveness), it can be used both to

clarify the targets of university courses for prospective teachers and to choose the most effective way to achieve them.

From a methodological perspective, applying this model allows us to view the preparation of prospective teachers not only as a process of knowledge acquisition, but also as the development of a holistic system of professional thinking that integrates various levels of teaching experience. The application of the TPACK model provides a foundation for a systematic analysis of the interactions between content, methods, and teaching tools, which facilitates the development of scientifically based approaches to the design of educational programmes. Thus, the emphasis shifts from specific methodological approaches to identifying universal principles for organising the educational process in a digital educational environment.

The *aim* of this article is to demonstrate the effectiveness of the TPACK model as a conceptual framework for designing a university course that integrates proven educational practices. The course is intended to develop a comprehensive system of professionally significant knowledge, skills, and abilities in prospective teachers, exemplified by an elementary geometry course.

Literature Review

MKT structure

Designing university courses that train prospective teachers in the field of science related to their professional field requires a clear understanding of the structure of subject-specific pedagogical knowledge necessary for teaching. For example, for mathematics teachers, this structure is the MKT (mathematical knowledge for teaching). The MKT structure was empirically derived by D. L. Ball, M. H. Tames, and G. Phelps [3], as well as H. Cheek and K. Beswick [4], based on observations conducted during mathematics lessons. The subject component of such knowledge – Subject Matter Knowledge (SMK) includes: Common Content Knowledge (CCK) – mathematical knowledge in itself, not integrated with teaching mathematics; Specialised Content Knowledge (SCK) – the knowledge of mathematics that is important in its teaching, for example, the knowledge of how to present a mathematical fact in a form suitable for students' perception or to explain to them what their mistake is related to when solving a problem without reducing the level of scientificity; Horizon Content Knowledge (HCK) – knowledge that allows us to see the relationship between different topics and sections of the mathematics course, to highlight the fundamental ideas of a particular content. Such knowledge helps, for example, to correctly organise the updating of previously studied material in the lesson, to focus the attention of students on a particularly significant fact or method.

The MKT also included the pedagogical domain (Pedagogical Content Knowledge – PCK), previously described by L. S. Shulman [9] which is linked to the mathematical content defined by the three components presented below. Knowledge of Content and Teaching (KCT) combines knowledge of teaching and knowledge of mathematics, that is, in fact, represents the field of knowledge of didactics in relation to the subject context, for example, knowledge of what methods, techniques

and strategies will be effective in organising students' acquisition of specific mathematical content (theorems, formulas, methods for solving problems), as well as for objectively assessing the results of its acquisition, what didactic materials are appropriate to use in this case. This component is directly related to the SCK component.

The Knowledge of Content and Students (KCS) component combines knowledge about students and knowledge of mathematics and, for example, allows the teacher to predict in advance, to anticipate what difficulties a student may experience when studying a particular material, what mistakes he/she may make when solving a particular problem, and, on the contrary, what can be interesting and motivating in the content of the topic. Also, the knowledge related to this component can help the teacher correctly interpret the not fully logical and coherent oral statements and written works of students. Thus, this component is aimed at using psychological and pedagogical knowledge about the age and personal characteristics of students in a subject context.

The third component of PCK – Content and Curriculum (KCC) – is the knowledge of standards and curricula, how they present the subject content, how to build learning goals, and what the mandatory learning outcomes should be for each section of the school course.

The analysis of scientific papers by R. Delgado-Rebolledo and D. Zakaryan [10], A. Dreher, A. Lindmeier, A. Heinze et al. [11], and M. Koponen, M. Asikainen, A. Viholainen et al. [12], which discuss the components of teachers' knowledge for teaching mathematics and their role in enhancing the effectiveness of the educational process across various contexts, not only provides insight into the composition and structure required of such knowledge but also allows for the generalisation of its essential properties. The primary properties can be summarised as follows:

- functional completeness, i.e. sufficiency for the high-quality performance by the teacher of work functions related to the subject area;
- a variety of connecting lines in a single structure of knowledge MKT, determining active mutual influence on each other;
- activity-based and procedural nature, which determines the chain of knowledge development: “I know what – I know why – I know how”, thanks to which knowledge is expressed in the ability to carry out activities; and
- the presence in the structure of knowledge about knowledge and knowledge about ignorance (reflected forms of knowledge), which direct the teacher to the need for the constant development of their own knowledge system.

We would like to add that as a characteristic of the MKT structure itself, it would be worth mentioning adaptability as the ability to respond to changes in its individual components, taking into account their characteristics. Another of its properties, in our opinion, is universality, which is determined by the possibility of its application, precisely as a structure, to other subject areas. Examples of such knowledge structures from other scientific fields, described in scientific and methodological literature, may be the following: Science Knowledge for Teaching (SKT) – focuses on

understanding scientific concepts, common student misconceptions, and explanation features, and is generally related to natural sciences; Language Knowledge for Teaching (LKT) is a set of knowledge for language teachers that includes knowledge of grammar and linguistic structures, knowledge of how children acquire language, knowledge of ways to explain linguistic phenomena and work with typical errors; an actively researched framework is Computing / CS Knowledge for Teaching (CSKT) or PCK for Computing, which relates to the knowledge necessary for teaching in the field of information technology. Thus, the structure of subject-pedagogical knowledge for teaching can serve as a conceptual framework for designing the professional training of future subject-area teachers.

The TPACK model and its application to the context of teaching mathematics

Since technology is now perceived as an integral part of education, the knowledge that any teacher should have should include technological knowledge, including knowledge about tools and smart technologies, as well as possible ways to use them wisely and effectively. In this regard, P. Mishra and M. L. Kohler [8] had a need to turn to the TPACK model as a theoretical basis for studying the knowledge of teachers necessary for integrating technologies into their teaching practice. Most articles mentioning TPACK are published mainly in technology journals and often focus on this particular component of the knowledge of prospective teachers. However, as rightly noted by Y. Morales-López, Y. Chacón-Camacho, and W. Vargas-Delgado [13], even if the teacher has extensive knowledge in the field of technology, this does not give him/her the opportunity to achieve the correct integration of ICT into educational spaces.

Studying the three main components of knowledge – Content Knowledge (CK), Pedagogical Knowledge (PK), and Technological Knowledge (TK) – across various contexts and subject areas, S. Cox and C. R. Graham [14], as well as W. Zhang and J. Tang [15], describe their possible paired combinations. They conclude that the core of the TPACK model, which integrates all these components, provides insight into the knowledge required for a modern teacher. In this regard, the researchers emphasise that this framework can primarily inform the training of prospective teachers.

Considering the TPACK model in the context of teaching mathematics, particularly geometry, researchers Y. Morales-López, Y. Chacón-Camacho, and W. Vargas-Delgado [13], as well as C. Soldano, Y. Luz, F. Arzarello et al. [16], rightly argue that technological knowledge cannot be considered in isolation from the subject content and didactics. Technological resources facilitate the assimilation of mathematical concepts by providing clarity, helping to identify patterns, establishing mathematical connections, generalising statements, and organising knowledge control. However, all these processes shape the mathematical activity itself, with technology serving only as a tool. In this regard Y. Morales-López, Y. Chacón-Camacho, and W. Vargas-Delgado [13], D. Apriandi, H. Retnawati, and A. M. Abadi [17] use the term “technological mathematical knowledge” (TMK) to describe the three-component knowledge system. When describing the structure of the TPMK,

the researchers determine the place for the MCT in it and consider it as a contextual specification of the TPACK.

In addition, researchers emphasise that the use of technology in education should be reasonable, relevant, and appropriate. In mathematics education, none of these requirements can be fulfilled unless the teacher's strong knowledge of technology is complemented by a thorough understanding of the subject matter and its didactics. At the same time, F. K. Bonafini and Yu. Li [18], M. V. Derbush and S. N. Skarbych [19], as well as E. N. Shutenko and A. I. Shutenko [20], rightly note that technology alone is insufficient to significantly improve learning. Moreover, being carried away by technologies, one can distort a mathematical fact, replace a rigorous mathematical proof with visualisation, or not take into account how specific students perceive a particular mathematical concept. Therefore, the university training of mathematics teachers in the field of school geometry should ensure that they master the entire set of mathematical knowledge on their own, as well as technological and pedagogical knowledge of mathematical content. The integration of these three elements gives a systemic effect in the teacher's readiness to carry out their professional activities in a high-quality manner.

Thus, the integrative TPACK model – which encompasses mathematical knowledge, mathematical knowledge for teaching, and technological pedagogical knowledge of mathematical content – can serve as a guide for identifying the specific learning outcomes of university courses in elementary mathematics for prospective teachers, as well as for determining the most effective educational practices to achieve these outcomes.

Although the analysis is conducted within the context of mathematics teaching, the propositions presented reflect broader patterns in the construction of pedagogical knowledge. The TPACK model is essentially a universal methodological framework applicable to research on the integration of content, pedagogy, and technology across various disciplinary fields. This approach facilitates the formation of a unified theoretical framework for the study, design, and development of teacher professional competence, regardless of subject specialisation.

Methodology, Materials and Methods

The study is based on the comprehensive use of theoretical and empirical methods, integrated into a unified methodological framework grounded in the conceptual tenets of the TPACK (Technological Pedagogical and Content Knowledge) model. The theoretical study aimed to align our research perspective with the contemporary scientific understanding of the structure of subject-pedagogical knowledge and its role in the development of teachers' professional competence. Another objective, achieved through the analysis of scientific and methodological literature, was to identify and systematise educational practices whose use in the professional training of prospective teachers ensures the holistic development of subject, pedagogical, and technological components of knowledge and facilitates their integration within the context of the TPACK model. This research design allowed us to

relate the phenomenon under study – the development of integrative knowledge in prospective mathematics teachers – to the broader context of teacher training, which involves the interrelationship between content, pedagogical strategies, and teaching technologies.

The results of the analysis of scientific sources were compared with the study's objectives through discursive reflection, which serves not only as a method of understanding but also as a means of theoretically reconstructing the experience of teaching. This approach, combined with an analysis of mathematics teachers' own teaching experiences, enabled us to develop an updated scenario for an elementary geometry course as an experimental model for implementing subject-specific pedagogical knowledge.

The experimental portion of the study aimed to identify the impact of an elementary geometry course on the development of the integrative component of knowledge in prospective mathematics teachers. The TPACK model, adapted to the study's objectives, was used as a measurement instrument. To assess and self-assess the knowledge of future teachers, the instruments developed by Y. Morales-López, Y. Chacón-Camacho, and W. Vargas-Delgado [13], as well as M. Schmidt, E. Baran, A. D. Thompson et al. [21] were used and modified. The questionnaires were translated into Russian and Kazakh and adapted to the context of teaching elementary geometry. The assessment was conducted using a five-point scale, where 1 = novice, 2 = beginner, 3 = advanced beginner, 4 = competent, and 5 = expert. Categories related to content knowledge were refined to take into account the specifics of the geometric material.

The survey was conducted twice: once before and once after the course. A total of 56 third-year university students participated in the study. All participants were informed about the study's objectives and content and provided their consent.

To obtain an objective assessment of the development of prospective teachers' knowledge systems, pre-test and post-test evaluations were conducted. During these assessments, students were given tasks consisting of three components: subject matter, technological, and methodological.

In the subject matter component, prospective teachers were offered problems in which the geometric figure was not explicitly named but was only specified by its description. The following task can serve as an example.

Problem. In a convex quadrilateral ABCD it is known that $AB \parallel CD$, $BC = AD$, and the diagonals AC and BD intersect at point O, with $\angle BAO = \angle DCO$. Determine the type of quadrilateral ABCD and prove your conclusion. Indicate which defining properties (criteria) of this type of quadrilateral you used in the proof. Compile a maximally complete list of properties of this figure.

In the technological component of the task, students were asked to: 1) construct in the GeoGebra environment a dynamic model whose construction was based on the definition or a characterising property of the figure under consideration in the problem; 2) use the tools of the dynamic environment to identify the properties of the given figure; 3) investigate possible “degenerations” of the model or bound-

ary cases. The methodological component required students to design a scenario of school students' learning activities for working with the proposed problem on the basis of the dynamic models they had constructed.

Each component of the task was scored on a 100-point scale. According to the results of their performance on each component, students were assigned to the same levels as in their self-assessments: Level 1 (0–24 points), Level 2 (25–49 points), Level 3 (50–74 points), Level 4 (75–89 points), and Level 5 (90–100 points).

To test for statistically significant differences between the mean scores obtained before and after the course, the Student t-test was used.

During the course, systematic observation of students' activities was also carried out, and the subject-related learning outcomes for each module were routinely monitored by means of written tests; the dynamics of students' knowledge development were analysed and discussed. This set of assessment procedures ensured a methodological linkage between observation, self-assessment, and objective evaluation.

Results

Stage 1. Analysis of effective educational practices in the context of the TPACK model

When redesigning a university course for prospective teachers using the TPACK model, we initially assumed that effective teaching practices for each course within the teacher education programme could be supported by the same methods applied throughout university education as a whole. This assumption is reflected in the research conducted by T. V. Agapova and L. Yu. Aisner [22], R. M. Asadullin and O. V. Frolov [23], and M. Thurlings and P. den Brok [24]. Analysis of studies that theoretically and empirically substantiate the influence of the peculiarities of the organisation of the educational process on the results of training in various educational contexts allows us to identify effective educational practices (Table 1).

Table 1

Analysis of effective educational practices (extracurricular context)

Practice	Authors	Characteristics of educational practice	Possible problems and difficulties in the implementation of educational practice	Conditions for overcoming problems and difficulties
Collaboration and interaction in the learning process	T. V. Agapova, L. Yu. Aisner [22]; N. A. Astashova [28]; R. Hofmann, K. Ruthven [29]; J. Taggart, L. B. Wheeler [30].	– is most often implemented in the format of group or paired work; – contributes to the creation of a complex and relevant learning environment for the students themselves; – develops critical thinking, conscious acquisition of knowledge, and argumentation skills; – refers to the form of autonomous learning, which forms self-regulation and self-efficacy of students; – can be used in the study of many topics of mathematics; – is a significant means of developing intellectual qualities.	– teachers do not always have a sufficient understanding of the format and functions of dialogic interaction; – due to the unpreparedness of students, the implementation of dialogic educational interaction is often superficial; – dialogical learning is a significant departure from the typical practice of teaching some disciplines, such as mathematics.	– the dialogic form of education should be the object of study of prospective and practicing teachers; – active support of dialogue and interaction on the part of the teacher is necessary; – dialogic learning should be thought out by the teacher and provided with the necessary didactic materials.
Inquiry-Based and Problem-Based Learning (PBL)	A. Dzundza, I. Moiseyenko, V. Tsapov [31]; M. Juniantari, N. S. Degeng, S. Ulfa et al. [32]; A. Milner, A. Scholkmann [33].	– effective practice in higher education; – focuses on five main strategies, coordinated with each other: structuring and systematising knowledge for better application, maintaining the practice of effective student reasoning, developing self-study skills, teamwork and supporting student motivation by giving learning a meaningful goal; – is often crucial in the field of pedagogical education; – contributes to the formation of problem-solving skills, including those existing in real practice, metacognitive skills, creative thinking; the development of interest in learning and creative thinking.	– it is difficult to find interesting problem or research tasks related to a specific subject content; – not every teacher has the skills to organise research and problem-based learning; – most students are often not ready for independent research or solving a problem situation.	– you cannot limit yourself to a textbook when preparing research and problem tasks, it is important to refer to additional sources; – it is useful if the teacher has his own experience of participating in research and problem-based learning practices; – in the process of performing educational research or solving a problem situation, it is important to provide students with differentiated assistance, taking into account their capabilities; – it is necessary to analyse and discuss the mistakes of students in solving research problems or educational problems.

Practice	Authors	Characteristics of educational practice	Possible problems and difficulties in the implementation of educational practice	Conditions for overcoming problems and difficulties
Use of technology	D. Hillmayr, L. Ziernwald, F. Reinhold et al. [34]; G. B. Shabbat, A. L. Semenov [35]; I. B. Shmigirilova, S. V. Kollisnichenko, O. V. Grigorenko [36]; D. Thurm, B. Barzel [37].	– the learning process becomes more dynamic; – positive impact on students' academic performance, as well as on their academic motivation; – there are technologies that are more suitable for achieving learning goals in different disciplines; – provides an interactive environment, working with which you can visualise mathematical objects and their relationships; – allows conducting educational research, putting forward and testing hypotheses; – performing the function of scaffolding, it contributes to the development of a deeper understanding of the subject content among students; – forms digital competence in students in the context of didactics.	– a mismatched representation of modelling leads to false assumptions about the facts; – the use of technologies will not be significant if their potential is not didactically realised.	– the use of technology should be carefully thought out by the teacher in order to accurately and clearly represent the features of the subject content; – the teacher needs to constantly develop knowledge about modern technologies that can be used for teaching, as well as improve the skills of interacting with technologies.
Approximation of practice	H. Howell, J. N. Mikeska [38]; E. Kazemi, H. Ghousseini, A. Cunard et al. [39]; L. A. Trubina, E. L. Erokhina [40].	– is based on the assertion that teachers tend to teach the way they were taught; – is implemented in two versions: 1) the future model of professional practice is based on the teaching strategies that preservice teachers themselves learned, and 2) providing future teachers with the opportunity to rehearse situations that may arise in their future professional activities, for example, through peer training; – is aimed directly at the formation of practical professional skills and abilities in future teachers.	– teachers of pedagogical universities do not always use the most appropriate and effective teaching strategies; – organising training where students play the role of teachers requires a significant investment of time.	– to develop the professional competencies of teachers who train prospective teachers; – to introduce methodological strategies that will be useful to them in their professional activities as early as possible in the training of prospective teachers; – modelling of prospective professional activities should start with individual private situations, gradually integrating them.

Practice	Authors	Characteristics of educational practice	Possible problems and difficulties in the implementation of educational practice	Conditions for overcoming problems and difficulties
Ensuring academic motivation	Yu. N Koshnikova, E. A. Avdeeva [41]; L. Matos, W. Lens, M. Vansteenkiste et al. [42]; S. Senko, B. Dawson [43].	– an effective way that affects the educational process and the result of training; – two types of goal settings that determine the differences in students' motives: mastery goals and performance goals; – there is a positive correlation between students' motivation, their self-efficacy, and academic performance with the perceived value of the task.	– motivation is directly related to the individual preferences of students and the nature of educational activities.	– it is necessary to use active teaching methods; – the teacher should take care to increase the perceived value of educational tasks, taking into account the individual needs of students.

Each of the identified educational practices can be used in the process of training a prospective teacher, regardless of the subject area, but the subject context can play a significant role in the implementation of the practice. For example, K. Küçük and Ö. Gün [25], A. S. Rvanova [26], and E. V. Sukhodolova [27], when considering the use of technology in teaching mathematics – particularly geometry – focus on dynamic environments. One of the most popular is GeoGebra, the use of which makes it possible to dynamically present various mathematical facts. At the same time, it should be emphasised that the fundamental condition for the success of the use of technology is the focus on modelling the type of activity inherent in mathematics, as well as an adequate representation of mathematical facts.

It should be noted that each subject area has a specific set of pedagogical practices developed in the context of the relevant school course and proven effective in the educational process. These practices represent an integral part of subject-specific pedagogical experience and must be considered when designing and implementing university courses aimed at training prospective teachers. Below, we briefly describe those that have proven most effective in teaching mathematics.

Solving mathematical problems is one of the main activities of students in mastering mathematical courses. Numerous studies reveal the role of mathematical problems in the formation of knowledge, skills and thinking of students. Since problem solving requires knowledge and skills in applying a variety of strategies, you can learn to solve problems only by constantly practicing. Many researchers have repeatedly proved that the activity that is associated with solving problems is not only a means of assimilating the subject content, but also a means of forming the cognitive experience of students and their thinking. A. M. Choban-Piletskaya [44], I. B. Shmigirilova [45], Ç. Toraman, Ş. Orakcı, and O. Aktan [46] emphasise that the use of tasks aimed at various solution strategies, practice-oriented tasks, joint search for ideas and solution strategies, their discussion and selection of the most rational strategy is an additional development tool.

Formation of intra-mathematical connections. The ability to establish mathematical connections is recognised by researchers N. V. Kononenko, J. S. Tokareva, and P. A. Chukhriy [47], as well as C. A. Rodríguez-Nieto, V. Font Moll, V. Borji et al. [48], as an indicator of a deep understanding of mathematical content. Additionally, according to A. Beecher, M. Capraro, and Y. Lee [49], it can serve as a foundation for mathematical creativity. Understanding mathematical relationships is also often crucial in the problem-solving process. Having started working at school, today's students will be forced to take care of the formation of intra-mathematical connections among schoolchildren. Therefore, the training of prospective teachers at the university should also ensure the formation of these connections. The mechanism that allows the formation of intra-mathematical connections is active learning, joint reasoning, guided by the teacher with the help of questions.

A significant contribution to the professional and mathematical development of prospective mathematics teachers can be made by *analysing students' decisions and mistakes*. The thoughtful use of student errors as a "springboard" for further learning and deeper understanding is recognised as an effective practice in research conducted by S. V. Artemyeva and T. Kuryakova [50], V. A. Dalinger [51], and M. Shaughnessy, R DeFino, E. Pfaff et al. [52]). The deliberate error strategy develops students' attention and self-assessment skills. Analysis and discussion of mistakes in the training of prospective mathematics teachers implements a number of functions. Firstly, the student's understanding of the nature of the error and the reasons behind it leads to a deeper comprehension of mathematics. Secondly, discussing their own mistakes or those made by other students fosters a more positive attitude toward errors. Students cease to be ashamed and afraid of them, and, therefore, become more active in learning. For future mathematics teachers, this is also important because in their future professional activities they will be able to teach schoolchildren not to be afraid of making mistakes. Thirdly, by gaining experience in making and correcting mistakes, prospective mathematics teachers will gradually improve their skills in the constructive use of mistakes: they will learn to identify and understand what gaps in their learning content exist, and to find more precise ways to overcome these gaps.

In the context of the problem under consideration, the *method of local axiomatisation*, also known as the *method of small theories*, is relevant. This method is presented by G. Freudenthal [53] and A. A. Stolyar [54] through a sequence of steps:

- identification and formulation of the properties of the studied geometric situation;
- identification of deductive relationships between these properties;
- selection of a system of axioms, formulation of equivalent definitions; and
- study of various options for constructing a local axiomatic theory of a given geometric situation.

A. A. Stolyar [53] believes that local axiomatisation can be used as a special method of teaching mathematics, when students will be involved in educational research on the construction of "small theories". Agreeing with the author, we note

that in such studies it is appropriate to use dynamic models created in the GeoGebra environment, since the basis for their construction is a set of characteristic properties of the geometric situation, which is a local system of axioms.

Thus, the analysis of educational practices, conducted in the context of training prospective mathematics teachers, served not only as a means of identifying effective strategies for implementing an elementary geometry course but also as a methodological tool for understanding the principles of integrating content, pedagogy, and technology in professional education. This allowed us to substantiate an approach to course design focused on developing students' holistic knowledge of subject matter, pedagogy, and technology.

Stage 2. Design of the elementary geometry course

The choice of an elementary geometry course for the practical implementation of the TPACK model was motivated by the fact that geometry remains one of the most challenging areas for both current schoolchildren and prospective mathematics teachers. The course was thematically divided into modules. Each module was implemented in stages.

Stage 1 was conducted using a flipped learning format: students independently reviewed the section using school textbooks and materials provided by the teacher. The teacher also recommended digital learning resources (DLR), which the students used to update the subject content of this section of school geometry (e.g. beyim.ai, bilimland, twig-bilim, onlinemektep, urok.1c., euclidean.xyz, etc.). The stage is aimed at solving a number of problems. Firstly, the material already familiar from the school course is repeated, and gaps in knowledge in this section of elementary geometry are filled. Secondly, students study not only the content of the section as such, but also how this content is presented in school textbooks, that is, in essence, the formation and development of the CCK and SCK knowledge areas. Thirdly, on the basis of materials developed by the teacher, students supplement and systematise their knowledge, fixing them in the format of a set of sentences that determine the identified properties of a particular geometric situation (figure). That is, in fact, the first stage of local axiomatisation is implemented. Finally, it provided students with the opportunity to systematically familiarise themselves with available digital educational resources containing content related to the module.

At the *2nd stage*, after mutual verification of the identified properties, prospective teachers, in the course of a logical experiment, examine the resulting set of sentences to establish the possibilities of its logical organisation. Students are faced with a problem: from which minimal sets of sentences (systems of local axioms) that are part of the resulting set, all other sentences follow? Prospective teachers should receive verbal formulations of statements, prove statements or refute them, citing a counterexample, and conclude whether the statement is true or false. The work is completed by the formulation of alternative definitions of the figure under consideration or the features of the geometric situation. Such work is initially organised as a joint discussion under the direct guidance of the teacher. In the future, it is possible to organise independent work of students in pairs or small groups.

Thus, the component of mathematical knowledge of the SCS is updated and deepened, and in the process of systematisation, intra-mathematical connections are formed, and the knowledge component of the HCK develops.

Stage 3 is built as work with dynamic models. Students, working in the GeoGebra environment, construct a dynamic model of a geometric situation (figure) based on the definition. Then, by giving the dynamics of the model and performing the necessary measurements, they reveal the properties of the figure. Note that, when working with a dynamic drawing, students implement an inductive study, the conclusions of which require proof. It is imperative to draw the attention of students to this and organise work on proving the identified properties. Further, working in groups, students develop a scenario of how, using this model, to demonstrate to students a set of features of a geometric situation (figure), which together represent a feature of this figure.

The next task is to build a dynamic model of the geometric situation based on the obtained feature, to check the possibility of its use to demonstrate the rest of the set of properties of the shape. If there is time, it is also useful to conduct an additional study of the model to answer the question: does the model always exist in dynamics, and under what conditions does it degenerate. The stage ends with a presentation of the results of the work of each small group and their collective discussion.

This stage also solves several didactic problems: students once again repeat the properties and features of the studied geometric situation (figure), developing the corresponding components of knowledge (CCK and HCK); students form special content knowledge (SCK), which can be used in the future when teaching this section to schoolchildren; independent work of students in the GeoGebra environment and the inclusion of this technology in contextual teaching contributes to the formation of technological pedagogical knowledge of mathematical content (TPMK); working in pairs or small groups, as well as public presentation of the results, requires students to be able to interact with each other, argumentation skills that will allow them to convey their knowledge to others; in addition, by presenting the results of their work, students in some way approximate future professional practice.

At the *4th stage*, students solve problems, and also independently compose them. Before the lesson, students are asked to compose problems using the already learned properties and features of the geometric situation. These can be problems for proof, for example, similar to those that arose at the stage of the logical experiment, as well as new problems for proof and calculation. Then, in the classroom, students, working in pairs, exchange tasks. Each solves the problems compiled by another student, and then they discuss the solutions together. If an error is detected, they collaboratively determine the cause. At the end of the lesson, students receive sets of tasks from the teacher for independent solution. For example, the tasks provided in the school textbook on this topic can be utilised.

Stage 5 is built as a generalising lesson on the module. Various formats of such a lesson were tried. For example, future teachers working in groups were asked to cre-

ate mind maps reflecting the material being studied and the connections between individual facts. The Miro online platform was used to construct and visualise mind maps. Its key advantage is its support for synchronous, real-time collaboration with an unlimited number of participants, which is particularly valuable for the collective generation and structuring of ideas. As part of the summary lesson for each module, an analysis of digital learning resources (DLR) related to the module's topic was also organised. These were assessed based on their methodological validity, the quality of content presentation, and their potential effectiveness in school teaching. It is useful to end the class with a collective reflection on what important things the students have learned, what beliefs and skills they have acquired.

Thus, the course is largely focused on the independent work of students, which is supported by special educational practices, teaching materials and instructions developed by the teacher, as well as, if necessary, teacher consultations.

Stage 3. Results of the implementation of the new design of the elementary geometry course

The results of the implementation of the new course scenario were experimentally verified during a two-time survey (Table 2). The analysis of the results obtained indicates that there is a tendency for the development of the TPACK of prospective mathematics teachers.

Table 2

Results of self-assessment by prospective teachers of mathematics knowledge in accordance with the TPACK model

TPACK components	Before the course		After the course		Em-pirical value of Stu-dent's t-test
	M (mean value)	SD (stan-dard deviation)	M	SD	
Technology Knowledge (TK)	3.01	1.05	3.44	0.88	
TK1. I can easily master the technology	2.71	1.20	3.07	1.04	5.528
TK2. I follow important new technologies	3.36	0.90	3.96	0.57	7.291
TK3. I often experiment with technology	2.43	0.95	3.00	0.71	6.806
TK4. I know different technologies	3.32	0.81	3.75	0.74	6.423
TK5. I have the technical skills to use technology	3.25	1.03	3.39	0.87	2.406
Content Knowledge (CK)	3.35	1.07	3.87	0.90	
CK1. I I know the definitions of various geometric shapes studied at school	4.11	0.68	4.54	0.50	4.690
SK2. I can formulate the properties and characteristics of different shapes and classify them	4.00	0.66	4.39	0.56	5.966
CK3. I can prove theorems of school geometry	2.54	0.87	3.04	0.79	5.442
CK4. I can solve geometric problems	2.75	0.96	3.50	0.74	7.086
Pedagogical Content Knowledge (PCK)	2.22	1.09	2.83	0.94	
PCK1. I have an idea of the scope and stages of teaching school geometry topics	2.86	1.03	3.46	0.69	6.691

TPACK components	Before the course		After the course		Em- pirical value of Stu- dent's t-test
	M (mean value)	SD (stan- dard devi- ation)	M	SD	
PCK2. I know how to choose effective teaching approaches to guide students' thinking in geometry learning	1.96	0.79	2.57	0.74	6.219
PCK3. I am familiar with the common mistakes that students make in understanding various geometry topics	1.86	0.92	2.93	0.60	11.295
PCK4. I will be able to apply information related to real life when developing geometry activities	2.21	1.30	2.36	1.21	3.028
Technological Content Knowledge (TCK)	2.46	0.97	3.21	0.81	
TCK1. I know about technologies that can be used to study geometry	2.18	1.11	3.36	0.60	9.420
TCK2. I can interpret the basic properties of shapes using software	2.29	1.00	3.36	0.69	13.392
TCK3. I can use animation and graphics to enrich my proposal for mathematical activities	2.00	0.66	2.68	0.58	5.664
TCK4. I have the skills to use software installed on a computer for mathematical purposes (for example, Excel, GeoGebra, etc.)	2.25	0.58	2.61	0.59	4.340
TCK5. I can search the Internet for reliable information related to geometry	3.64	0.72	3.96	0.34	4.430
TCK6. I can highlight the advantages and disadvantages of using technological resources for teaching geometry	2.39	0.62	3.29	0.43	11.894
Technological Pedagogical Knowledge (TPK)	2.31	0.98	3.44	0.83	
TPK1. I can choose technologies that improve learning	1.75	0.51	2.79	0.68	10.507
TPK2. Studying at the university made me think more deeply about how technology can affect learning	2.93	1.32	4.25	0.69	10.149
TPK3. I think critically about how to use technology in teaching the subject	2.68	0.66	3.57	0.63	16.199
TPK4. I will be able to adapt the use of the technologies I am studying to my pedagogical activities	1.89	0.62	3.14	0.52	13.476
Technological Pedagogical Content Knowledge (TPCK)	1.85	0.83	2.77	0.87	
TPCK1. I have knowledge of relevant geometry teaching technologies, how to use them, and their impact on the learning process	1.46	0.57	3.18	0.72	18.166
TPCK2. I will be able to plan and conduct geometry classes that correctly combine mathematical knowledge, technology and approaches to learning	2.11	0.87	2.89	0.78	8.647
TPCK3. I will be able to choose technologies that will improve the form of presentation of mathematical content and my methodology for teaching mathematics	2.32	0.86	3.04	0.69	10.092
TPCK4. I know how to use technologies to identify the difficulties that students encounter when learning geometry.	1.50	0.63	1.96	0.74	6.904

Before the course, the average scores on the Technology Knowledge (TK) subscale were 3.01, on the Content Knowledge (CK) subscale 3.35, on the Pedagogical Content Knowledge (PCK) subscale 2.22, Technological Content Knowledge (TCK) 2.46, Technological Pedagogical Knowledge (TPK) 2.31, and Technological Pedagogical Content Knowledge (TPCK) 1.85. At the same time, the highest scores on the Technology Knowledge subscale can be explained by the trends of technology penetration into all spheres of human life and the conviction of pre-service teachers, perhaps not entirely justified, that they are ready to use technology; on the Content Knowledge (CK) subscale, the scores can be explained by the confidence of students in their own mathematical training, formed during their schooling. The lowest indicators are found in the subscales related to pedagogy, which is explained by the insignificant pedagogical experience of students or its absence.

After studying the course, one can trace a significant increase in the indicator for the subscales Technological Content Knowledge (TCK) – 3.21, Technological Pedagogical Knowledge (TPK) – 3.44, Technological Pedagogical Content Knowledge (TPCK) – 2.77, which reflects the integrative component of this course. The statistical significance of the changes was confirmed using the Student's t-test. The values of t_{emp} for all values of the subscale indicators, except for the TK 5 indicator, are greater than $t_{cr} = 3.476$ at a significance level of 0.001. For the TK 5 indicator, the value of t_{emp} is greater than $t_{cr} = 2.004$ at a significance level of 0.05.

Thus, TK 5 demonstrated the least pronounced change compared to the other components, while remaining statistically significant. The initial self-assessment for this item ($M = 3.25$) was relatively high. This can be explained by the tendency of students to initially overestimate their technical skills due to insufficient experience in applying them in professional contexts. During the learning process, these perceptions are corrected through practical activities and feedback. As a result, the final self-assessment becomes more realistic, and the gain is more modest compared to those competencies whose initial level was objectively lower.

Table 3 presents the results of the statistical analysis of the task performance before and after the course.

Table 3

Results of statistical processing of data on task completion before and after the course

TRACK components	Before the course		After the course		Empirical value of Student's t-test
	M (mean value)	SD (standard deviation)	M	SD	
Technology Knowledge (TK)	2.89	0.82	3.46	0.74	8.000
Content Knowledge (CK)	2.68	0.88	3.88	0.76	18.531
Pedagogical Content Knowledge (PCK)	2.18	0.86	3.27	0.75	17.062

The analysis of the results indicates a positive trend in the development of an integrative knowledge system among prospective teachers: the empirical t-value

(t_{emp}) for each knowledge component exceeds the critical value ($t_{cr} = 3.476$) at the 0.001 significance level. The standard deviation decreased, indicating greater stability of the results.

It is also noteworthy that, by the end of the course, the performance on this comprehensive task was higher than the results of a traditional written test. This effect points to the operation of a systemic mechanism for integrating all components of professional knowledge, whereby the interaction of subject matter, procedural, and instrumental aspects creates an additional cognitive resource.

An expansion of students' methodological and technological knowledge is also evidenced by the fact that they gradually began to take the initiative in exploring and using, in addition to GeoGebra, other digital dynamic tools (e.g. Desmos, Cabri Geometry, etc.), integrating them into the process of completing the tasks.

Discussion

A methodologically significant result of the study is the confirmation of the effectiveness of the integrative approach underlying the TPACK model. This model not only serves as a tool for analysing the structure of professional knowledge but also provides a framework for designing an educational process focused on its development. From a methodological perspective, TPACK serves as a concept that facilitates the transition from describing individual components of professional training (content, pedagogical, and technological) to considering them holistically as dynamically interacting elements of a system. This understanding allows us to view the process of professional development of prospective teachers as a process of consciously integrating knowledge, skills, and methods of action developed in various educational contexts.

It should also be noted that the key element in the triad of knowledge is subject matter knowledge: knowledge of teaching methods and technological pedagogical knowledge cannot be developed without an adequate mastery of the subject content. The integration of knowledge is largely determined by its procedural, activity-based nature, which is manifested through professionally significant skills.

The analysis of educational practices conducted in the study by Yu. N. Koreshnikova and P. S. Sorokin [55] reveals that all of them are, to some extent, focused on self-regulated learning, which is based on specially designed educational situations. It is important that these situations are carefully planned, didactically supported, and that students' independent activities are, when necessary, accompanied by timely, thoughtful, and unobtrusive teacher assistance. It is also noteworthy that all practices are related to each other. For example, problem-based learning or solving a complex mathematical problem is difficult to implement without the participation of students in a joint discussion of the problem. Technologies, in turn, help in solving the problem and contribute to a deeper understanding of intra-mathematical relationships. At the same time, the problem is often impossible to solve without understanding the connections between mathematical facts. The analysis of stu-

dents' decisions and mistakes of their own and their classmates is one of the ways to approximate future professional practice.

Therefore, to improve the results of training prospective teachers, it is important to reasonably combine various educational practices that have already proven their effectiveness. An attempt to integrate such practices was implemented in the research process in the course of elementary geometry. An important characteristic of this course is that the TPACK model was taken into account in its design. In this regard, our study confirmed the previously expressed views of Y. Morales-López, Y. Chacón-Camacho, and W. Vargas-Delgado [13], as well as D. Apriandi, H. Retnawati, and A. M. Abadi [17], that this model can serve as a foundation for analysing the relevance of various types of students' cognitive activities when designing professional development programmes for teachers incorporating technology.

The focus of the course on the formation of an integrative set of knowledge of mathematics, mathematical knowledge for teaching and technological pedagogical knowledge of mathematical content in prospective mathematics teachers determined its organisation, thanks to which students, repeatedly reproducing various types of active and sufficiently independent educational activities, will be able to transmit the acquired experience to their students in the future. At the same time, the implementation of the course was quite difficult for both students and teachers, as it required a high level of student independence. In addition, the content (geometric) basis of the activity was not immediately absorbed by prospective teachers. Observation of students in the classroom indicated that the stage of independent repetition of the section on school textbooks at the beginning of the course was not always implemented qualitatively, which was reflected in further work. Gradually, the prospective teachers came to realise the importance of this stage, which contributed to an increase in the quality of its implementation. One of the most difficult stages was the logical experiment, which, in fact, was conducted in the format of educational research. In this regard, the teacher was required to carefully plan the lesson and thoughtfully consider its didactic support – those training materials and instructions that, without providing ready-made knowledge or solutions, would facilitate the students' activities to acquire knowledge and guide the discussion of the problem in the right direction.

As they mastered the course, discovering all the new details in each of the areas of TPACK knowledge and understanding the need to continue working on their mastery, on the contrary, they could be more critical of themselves. This idea is confirmed by the observation of students' activities throughout the course. Particularly noteworthy is the systemic effect observed from integrating all components of prospective teachers' knowledge, whereby subject matter, methodological, and technological skills are activated and function in a coordinated manner.

It is also worth noting that during the classes in elementary geometry, the significant potential of local axiomatisation as a method of teaching mathematics was confirmed. Further research in this area will be relevant, since the sources on the use of this very effective method in educational practice, especially in the situation of digitalisation of education, are clearly insufficient.

Thus, the results of our study complement the previously obtained data by J. A. Hall, J. Lei, and Q. Wang [56], as well as İ. Saralar-Aras and B. Birgili [57], demonstrating that training based on the TPACK model and incorporating a variety of active educational practices supports the development of an integrated knowledge set among students – prospective mathematics teachers. At the same time, our results demonstrate that the effectiveness of knowledge integration is determined not only by the educational technologies used but also by the specifics of the organisation of learning activities. This underscores the need for further methodological understanding of the design principles of university courses for prospective teachers as an open system, incorporating cycles of reflection, self-regulation, and experience reconstruction. With this approach, each course becomes not simply a means of knowledge acquisition but a model of professional activity, in which students master pedagogical methods, develop a research stance, and the ability to construct their own pedagogical solutions. Thus, the implementation of an elementary geometry course within the TPACK model demonstrates the potential of an activity-based approach as a methodological basis for developing professional competence.

Conclusion

Thus, the research conducted enabled us to formulate principles for designing a university course that ensures the holistic development of subject matter, pedagogical, and technological knowledge in prospective teachers:

- the principle of integrating three types of knowledge (Content, Pedagogical, and Technological): course design should ensure a conscious connection between subject matter, pedagogical strategies, and digital technologies, this requires the development of assignments and learning situations in which each TPACK component is not only presented but also interacts with others, creating a holistic professional and cognitive activity for students;
- the principle of activity-based integration: knowledge integration should occur not at the level of declarations, but through students performing actions that simulate the professional activity of a teacher;
- the principle of reflective self-regulation of learning: students should be involved in the processes of analysing and self-evaluating their own activities, understanding the role of each TPACK component in achieving educational goals;
- the principle of content contextualisation: educational material and assignments should be anchored in real or simulated professional contexts;
- the principle of variability and flexibility of educational practices: course design involves a combination of various forms of activity (individual, group, collective), which allows for multiple transitions between the roles of student, researcher, and teacher;
- the principle of technologically mediated activity: technology should be viewed not as an external tool, but as a component of professional action, enabling visualisation, communication, modelling, collaboration, and data analysis; and

– the principle of methodological reflection and transfer: the course should include stages of reflection on the pedagogical and technological approaches used, taking into account their applicability to other learning situations and contexts.

Methodologically, this result reflects the importance of moving from the fragmented acquisition of subject, pedagogical, and technological knowledge to their holistic integration in the process of teaching. This integration ensures not only the mastery of individual components of professional competence, but also the development of the ability to act in complex pedagogical situations, where the meaningful use of technology becomes part of professional thinking.

In this context, the methodological readiness of university teachers to implement practices based on the synthesis of subject content, technologies, and pedagogical strategies is particularly important. The effectiveness of the integrative training of prospective teachers depends on the level of their reflective and design competence. Thus, the study results not only clarify the understanding of the structure of teacher professional knowledge but also demonstrate the methodological potential of the TPACK model as a basis for designing new types of educational programmes.

The practical significance of this work lies in its specification of the principles for organising a university course that promotes the development of integrative knowledge. The developed and tested scenario for the elementary geometry course demonstrated that its methodological concept – the combination of logical and mathematical content, research activities, and technologically supported independent work – can be transferred to other disciplines and areas of study. Thus, the course serves as a model of activity-based and integrative learning, focused on developing the professional thinking of prospective teachers.

A promising direction for further research is the theoretical substantiation and empirical verification of pedagogical conditions that ensure the implementation of the TPACK model in the system of university courses for training prospective teachers as a holistic methodological structure of professional development.

The results presented in the study may have limitations, primarily due to the fact that the learning process and its results are influenced by many factors related to the personal characteristics and abilities of students, the training of university teachers, the learning environment, therefore, the implementation of such a scenario and its impact on the development of TPACK in prospective teachers in other contexts may differ slightly in one direction or another.

The sustainability of students' achievements can be assessed in a longitudinal study with delayed measurements (after the teaching practicum and 6–12 months of school employment) and a control group of novice teachers who did not complete the course. The same tool can be used to conduct this stage of the study.

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